

REF ID: A6110

NUSC Technical Report 8631  
6 October 1989



AD-A220 320

# Determination of Noise Field Directionality Directly from Spatial Correlation for Linear, Planar, and Volumetric Arrays

Albert H. Nuttall  
Surface ASW Directorate

DTIC  
ELECTED  
APR 11 1990  
S D  
CD



**Naval Underwater Systems Center**  
**Newport, Rhode Island / New London, Connecticut**

Approved for public release; distribution is unlimited.

### Preface

This research was conducted under NUSC Project No. A75215, Subproject No. R00N000, "Determination of Concentrated Energy Distribution Functions in the Time-Frequency Plane," Principal Investigator Dr. Albert H. Nuttall (Code 304). This technical report was prepared with funds provided by the NUSC In-House Independent Research and Independent Exploratory Development Program, sponsored by the Office of Chief of Naval Research. Also, this work was sponsored by the NUSC Special Projects Office, Code 01Y, under Job Order No. 701Y12.

The technical reviewer for this report was Dr. Roy L. Streit (Code 214).

Reviewed and Approved: 6 October 1989

  
Daniel M. Viccione  
Associate Technical Director  
Research and Technology

# REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

<b>1. AGENCY USE ONLY (Leave blank)</b>			<b>2. REPORT DATE</b> 6 OCT 1989		<b>3. REPORT TYPE AND DATES COVERED</b>		
<b>4. TITLE AND SUBTITLE</b> <b>DETERMINATION OF NOISE FIELD DIRECTIONALITY DIRECTLY FROM SPATIAL CORRELATION FOR LINEAR, PLANAR, AND VOLUMETRIC ARRAYS</b>			<b>5. FUNDING NUMBERS</b>  PR      A75215 amd 701Y12				
<b>6. AUTHOR(S)</b>  Albert H. Nuttall							
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b>  Naval Underwater Systems Center New London Laboratory New London, CT 06320			<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>  NUSC Technical Report 8631				
<b>9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b>  Office of the Chief of Naval Research, Arlington, VA 22217-5000 and NUSC Special Projects Office, Code 01Y			<b>10. SPONSORING/MONITORING AGENCY REPORT NUMBER</b>				
<b>11. SUPPLEMENTARY NOTES</b>							
<b>12a. DISTRIBUTION/AVAILABILITY STATEMENT</b>  Approved for public release; distribution is unlimited.			<b>12b. DISTRIBUTION CODE</b>				
<b>13. ABSTRACT (Maximum 200 words)</b>  The spatial correlation between two points of an array is given by a two-dimensional integral in terms of the noise field directionality. Depending on the dimensionality of the array, this integral equation can be partially solved, to yield explicit expressions for the noise field directionality in terms of a multi-dimensional Fourier transform. In particular, for a linear array, a one-dimensional collapsed field distribution can be determined; for a planar array, the sum of symmetrically-arriving rays can be solved for; and for a volumetric array, the complete field can be found. The effects of finite length and discrete arrays on the estimate of the noise field directionality are also considered.  ... about Arrays;							
<b>14. SUBJECT TERMS</b>  →Directionality; Spatial Correlation Array Processing			Linear Array; Planar Array; Volumetric Array Fourier Transform;		<b>15. NUMBER OF PAGES</b> 44		
<b>16. PRICE CODE</b>							
<b>17. SECURITY CLASSIFICATION OF REPORT</b>  UNCLASSIFIED		<b>18. SECURITY CLASSIFICATION OF THIS PAGE</b>  UNCLASSIFIED		<b>19. SECURITY CLASSIFICATION OF ABSTRACT</b>  UNCLASSIFIED		<b>20. LIMITATION OF ABSTRACT</b>  UL	

## TABLE OF CONTENTS

	Page
LIST OF ILLUSTRATIONS .....	ii
LIST OF SYMBOLS .....	ii
INTRODUCTION .....	1
CHARACTERIZATION OF NOISE FIELD .....	5
LINEAR ARRAY .....	9
Solution of Integral Equation .....	10
Example .....	12
Discrete Infinite-Length Array .....	13
Discrete Finite-Length Array .....	15
PLANAR ARRAY .....	17
Solution of Integral Equation .....	18
Behavior Near Plane of Array .....	20
Discrete Infinite-Length Array .....	21
Discrete Finite-Length Array .....	24
VOLUMETRIC ARRAY .....	25
Solution of Integral Equation .....	25
Simultaneous Equations .....	27
Angular Representations .....	29
SUMMARY .....	33
APPENDIX A. ALTERNATIVE LOCATION OF LINEAR ARRAY .....	35
APPENDIX B. ALTERNATIVE LOCATION OF PLANAR ARRAY .....	37
APPENDIX C. EXAMPLE FOR VOLUMETRIC ARRAY .....	39
REFERENCES .....	43

## LIST OF ILLUSTRATIONS

Figure		Page
1	Coordinate System	5
2	Window Function $Q^2$	28

## LIST OF SYMBOLS

$\theta$	polar angle, figure 1
$\phi$	azimuthal angle, figure 1
$f$	temporal-frequency
$N_f(\theta, \phi)$	noise field directionality, (1)
$\tau_1$	time of arrival at location $x_1, y_1, z_1$ , (2)
$c$	speed of propagation, (2)
$H_1(f)$	transfer function, (3)
$\lambda$	wavelength = $c/f$ , (3)
$x, y, z$	separation distances, (5)
$G_f(x, y, z)$	spatial correlation at temporal-frequency $f$ , (6)
$G_f(z)$	spatial correlation for linear array, (7)
$\bar{N}_f(\theta)$	integrated noise field directionality, (8)
$\delta$	delta function, (9)
$\Delta$	spacing in $z$ , (12)
$M$	size of fast Fourier transform, (13), (14), (29), (30)
$\oplus$	convolution, (15a)
$w(z)$	weighting, (15b)
$W(u)$	window, (15c)

$G_f(x,y)$  spatial correlation for planar array, (16)  
 $\tilde{N}_f(\theta,\phi)$  sum of symmetrically-arriving rays, (17)  
 $I(u,v)$  two-dimensional Fourier transform of  $G_f(x,y)$ , (18)  
 $\arg(z)$  argument of complex number  $z$ , (21)  
 $C_1$  circle of radius 1 at origin, (23)  
 $F$  auxiliary function, (24)  
 $\Delta_x, \Delta_y$  spacings in  $x,y$ , (28)  
 $N$  size of fast Fourier transform, (29),(30)  
 $I(u,v,w)$  three-dimensional Fourier transform of  $G_f(x,y,z)$ , (36)  
 $q(z)$  weighting in  $z$ , (36)  
 $Q(t)$  Fourier transform of  $q(z)$ , (37)  
 $s$  auxiliary function, (42)  
 $F_1, F_2$  auxiliary functions, (44)  
 $w_1, w_2$  two distinct values of  $w$ , (46),(47)  
 $Q_n(\pm)$  abbreviated notation, (48)  
 $D$  denominator, (49)  
 $L_z$  effective length of weighting  $q(z)$ , figure 2  
 $I(\pm)$  abbreviated notation, (55)  
 $\theta'$  complementary angle =  $\pi - \theta$ , (56)  
 $I'(\pm)$  abbreviated notation, (57)



Accession For	
NTIS CRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification _____	
By _____	
Distribution / _____	
Availability Codes	
Dist	Avail and/or Special
A-1	

DETERMINATION OF NOISE FIELD DIRECTIONALITY DIRECTLY FROM  
SPATIAL CORRELATION FOR LINEAR, PLANAR, AND VOLUMETRIC ARRAYS

INTRODUCTION

When an array is located in a homogeneous stationary noise field, measurement of the crosscorrelations between all pairs of separated elements, at each temporal-frequency of interest, is the most general second-order statistical information that can be extracted. These spatial correlations depend upon the directionality of the surrounding noise field, which is the primary quantity of interest here. Instead of beamforming the element outputs, for example, and trying to suppress the inherent sidelobes by proper weighting procedures, we want to avoid any pre-conceived notions about data processing and go directly from the spatial correlation to the noise field directionality in as direct and simple a manner as possible.

However, because the noise field directionality is a two-dimensional function of polar and azimuthal angles, some inherent loss or condensation of information takes place with a linear array and, to a much lesser extent, with a planar array. Nevertheless, we want to preserve and extract the maximum amount of information about the noise field directionality, consistent with the dimensionality of the array employed, and to minimize the amount of data processing required.

We begin by assuming the array to be an infinite continuous line in the one-dimensional case, and solve the integral equation for the integrated (or collapsed) noise field directionality, at each temporal-frequency, in terms of the spatial correlation along the line. Then, we discretize the line, so as to be an equi-spaced array, and determine the effect that this limitation has upon the estimated directionality. Finally, we investigate the smoothing that is caused by the practical requirement that any physical array must have finite length. Thus, the facts that the spatial correlation will never be available on a continuum, nor for infinite separations, are included in the analysis.

A similar procedure is pursued for the two-dimensional case, where the planar array is presumed to have equal spacings  $\Delta_x$  and  $\Delta_y$  in the x and y dimensions, respectively. Again, the aliasing effects are considered, as well as the limitation of having to employ a finite-size planar array. Finally, in the three-dimensional case, where the problem is overdetermined, a plausible and efficient procedure for collapsing the surplus information is presented, although it is recognized that an unlimited number of alternatives exist.

Although it was stated that the noise field directionality is of interest, this does not preclude the presence of plane-wave arrivals, that is, additive signals or interferences in the background. In fact, the examples are specifically of that type, for these can be considered as the fundamental building blocks of a general noise field.

Some related results on this problem of restoring the noise field directionality from the spatial correlation are given in [1,2,3,4], but limited to the line array. Specifically, [1] gave a least squares approach, starting from a discrete finite-length array. However, ill-conditioning of the simultaneous linear equations for the noise field directionality precluded its use for more than approximately ten elements. This ill-conditioning is circumvented here by deferring the discretization until after the integral equation is solved; this procedure for the line array was first given in [4].

## CHARACTERIZATION OF NOISE FIELD

Let  $N_f(\theta, \phi)$  be the intensity of the homogeneous stationary noise field at temporal-frequency  $f$ , arriving from direction  $\theta, \phi$ , where  $0 \leq \theta \leq \pi$ ,  $-\pi < \phi \leq \pi$ ; see figure 1. The amount of power received in solid angle  $d\theta d\phi \sin\theta$  about  $\theta, \phi$  is

$$d\theta d\phi \sin\theta N_f(\theta, \phi). \quad (1)$$

We call  $N_f(\theta, \phi)$  the noise field directionality; the product  $\sin\theta N_f(\theta, \phi)$  could be called the plane-wave density.

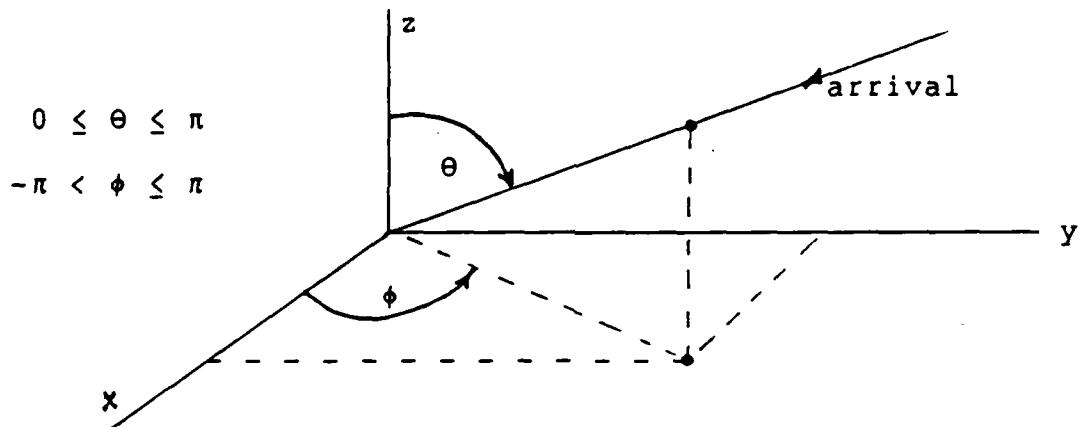


Figure 1. Coordinate System

Consider general field point  $x_1, y_1, z_1$ . Then if the time of arrival at the origin, of the component from direction  $\theta, \phi$ , is zero, then the time of arrival at  $x_1, y_1, z_1$  is

$$\tau_1 = -\frac{1}{c} (x_1 \sin\theta \cos\phi + y_1 \sin\theta \sin\phi + z_1 \cos\theta), \quad (2)$$

where  $c$  is the speed of propagation. Therefore, the transfer function at  $x_1, y_1, z_1$  applied to the arrival from direction  $\theta, \phi$  is

$$H_1(f) = \exp(-i2\pi f \tau_1) = \\ = \exp\left(i\frac{2\pi}{\lambda} (x_1 \sin\theta \cos\phi + y_1 \sin\theta \sin\phi + z_1 \cos\theta)\right), \quad (3)$$

where wavelength  $\lambda = c/f$ .

The elemental contribution to the crosscorrelation between this arrival at  $x_1, y_1, z_1$  and  $x_2, y_2, z_2$ , at temporal-frequency  $f$ , is then

$$d\theta d\phi \sin\theta N_f(\theta, \phi) H_1(f) H_2(f)^* = d\theta d\phi \sin\theta \times \\ \times N_f(\theta, \phi) \exp\left(i\frac{2\pi}{\lambda} (x \sin\theta \cos\phi + y \sin\theta \sin\phi + z \cos\theta)\right), \quad (4)$$

where separations

$$x = x_1 - x_2, \\ y = y_1 - y_2, \\ z = z_1 - z_2. \quad (5)$$

If the arrivals from different directions are uncorrelated, the spatial correlation (at frequency  $f$ ) between two points separated by  $x, y, z$  is then given by integrating over all angular space, ~

$$G_f(x, y, z) = \int_0^\pi d\theta \int_{-\pi}^\pi d\phi \sin\theta N_f(\theta, \phi) \times \\ \times \exp\left(i\frac{2\pi}{\lambda} (x \sin\theta \cos\phi + y \sin\theta \sin\phi + z \cos\theta)\right). \quad (6)$$

The problem of interest is: given spatial correlation  $G_f(x, y, z)$  versus  $x, y, z$  (or restricted slices of  $G_f(x, y, z)$ ), solve for noise field directionality  $N_f(\theta, \phi)$  (or smoothed versions of

$N_f(\theta, \phi)$ ). That is, invert integral equation (6) for noise field directionality  $N_f(\theta, \phi)$  or for whatever can be determined. There are three cases that must be distinguished, namely, linear, planar, and volumetric arrays.

## LINEAR ARRAY

It is most convenient mathematically to locate the line array along the z axis, that is,  $x = y = 0$ . Then the exponential in (6) is independent of  $\phi$ , and (6) reduces to\*

$$\begin{aligned} G_f(z) &= \int_0^{\pi} d\theta \int_{-\pi}^{\pi} d\phi \sin\theta N_f(\theta, \phi) \exp\left(i\frac{2\pi}{\lambda} z \cos\theta\right) = \\ &= \int_0^{\pi} d\theta \sin\theta \bar{N}_f(\theta) \exp\left(i\frac{2\pi}{\lambda} z \cos\theta\right), \end{aligned} \quad (7)$$

where

$$\bar{N}_f(\theta) = \int_{-\pi}^{\pi} d\phi N_f(\theta, \phi) \quad \text{for } 0 \leq \theta \leq \pi \quad (8)$$

is the integrated or averaged noise field directionality, and  $G_f(z)$  is the one-dimensional spatial correlation at separation  $z$  along the line, both functions evaluated at frequency  $f$ .  $G_f(z)$  is the only second-order function that can be measured (or estimated) from the line array, and  $\bar{N}_f(\theta)$  is the only field function that can be determined. There is no possibility of undoing the integration of (8); this is a mathematical representation of the inherent conical symmetry of response of a linear

---

\*The case where the line array is located on the x axis is treated in appendix A.

array. It is also one reason for choosing the line array to lie along the  $\theta = 0$  axis, since all the two-dimensional field information is conveniently collapsed into a one-dimensional function of  $\theta$  alone. See appendix A for the problems associated with choosing a different coordinate system.

#### SOLUTION OF INTEGRAL EQUATION

To solve integral equation (7) for noise field directionality  $\bar{N}_f(\theta)$ , consider the following:

$$\begin{aligned}
 & \int_{-\infty}^{+\infty} dz \exp\left(-i\frac{2\pi}{\lambda} u z\right) G_f(z) = \\
 &= \int_0^{\pi} d\theta \sin\theta \bar{N}_f(\theta) \int_{-\infty}^{+\infty} dz \exp\left(-i\frac{2\pi}{\lambda} z(u - \cos\theta)\right) = \\
 &= \int_0^{\pi} d\theta \sin\theta \bar{N}_f(\theta) \lambda \delta(u - \cos\theta), \tag{9}
 \end{aligned}$$

where  $\delta$  is the delta function. Now let  $t = \cos\theta$ , which is a one-to-one transformation for  $0 \leq \theta \leq \pi$ , to get

$$\begin{aligned}
 & \int_{-\infty}^{+\infty} dz \exp\left(-i\frac{2\pi}{\lambda} u z\right) G_f(z) = \lambda \int_{-1}^1 dt \bar{N}_f(\cos(t)) \delta(u - t) = \\
 &= \begin{cases} \lambda \bar{N}_f(\cos(u)) & \text{for } |u| < 1 \\ 0 & \text{for } |u| > 1 \end{cases}. \tag{10}
 \end{aligned}$$

That is,

$$\bar{N}_f(\text{acos}(u)) = \frac{1}{\lambda} \int_{-\infty}^{+\infty} dz \exp\left(-i\frac{2\pi}{\lambda} u z\right) G_f(z) \quad \text{for } |u| < 1, \quad (11a)$$

or

$$\bar{N}_f(\theta) = \frac{1}{\lambda} \int_{-\infty}^{+\infty} dz \exp\left(-i\frac{2\pi}{\lambda} \cos\theta z\right) G_f(z) \quad \text{for } 0 \leq \theta \leq \pi. \quad (11b)$$

Here,  $\text{acos}$  is the principal value inverse cosine function.

Compare (11b) with starting point (7).

Thus, given the spatial correlation  $G_f(z)$  for all possible separations  $z$  along the line array, the integrated noise field directionality  $\bar{N}_f$  is available via a single one-dimensional Fourier transform.

## EXAMPLE

An example is informative at this point. Let

$$\bar{N}_f(\theta) = \delta(\theta - \theta_0), \quad 0 < \theta_0 < \pi.$$

Then the spatial correlation is, from (7),

$$G_f(z) = \sin\theta_0 \exp\left(i\frac{2\pi}{\lambda} z \cos\theta_0\right).$$

Observe that as  $\theta_0 \rightarrow 0$  or  $\pi$ , that is, endfire of the line array, the strength of this quantity decays to zero, due to the  $\sin\theta$  term in the area element in (1). Substitution of correlation  $G_f(z)$  into (11b) yields noise field directionality

$$\bar{N}_f(\theta) = \sin\theta_0 \delta(\cos\theta - \cos\theta_0) \quad \text{for } 0 \leq \theta \leq \pi.$$

Now the delta function here is located at  $\theta = \theta_0$  and has area  $1/\sin\theta_0$ . Thus,  $\bar{N}_f(\theta)$  is  $\delta(\theta - \theta_0)$ , as it should be; however, the trigonometric form shows  $\bar{N}_f(\theta)$  as the product of two terms, the first of which tends to zero as  $\theta_0 \rightarrow 0$  or  $\pi$ , and the second of which has an area that tends to infinity as  $\theta_0 \rightarrow 0$  or  $\pi$ . This behavior will re-occur in the following investigations.

We have employed the following useful property above: if  $g(x)$  has an isolated zero at  $x_0$ , then in the neighborhood of  $x_0$ ,

$$\delta(g(x)) = \delta(g'(x_0)(x-x_0)) = \frac{1}{|g'(x_0)|} \delta(x - x_0).$$

That is, the area of the delta function at  $x_0$  is equal to the reciprocal absolute slope of the argument at  $x_0$ , if nonzero.

## DISCRETE INFINITE-LENGTH ARRAY

If samples of spatial correlation  $G_f(z)$  at increment  $\Delta$  in  $z$  are available, an approximation to (11a) is afforded, for  $|u| < 1$ , by

$$\bar{N}_f(\cos(u)) \approx \frac{\Delta}{\lambda} \sum_{n=-\infty}^{+\infty} \exp\left(-i\frac{2\pi}{\lambda} u \Delta n\right) G_f(\Delta n), \quad (12)$$

the right-hand side of which has period  $\lambda/\Delta$  in  $u$ . Since the integrated noise field directionality in (11a) is defined on an interval of length 2, that is,  $-1 < u < 1$ , aliasing will occur in approximation (12) unless  $\Delta < \lambda/2$ . Thus, the spacing  $\Delta$ , between samples of  $G_f(z)$ , must be less than a half-wavelength at the temporal-frequency  $f$  of interest. This is presumed true henceforth.

Now if  $u$  is restricted to the values

$$u_m = \frac{m}{M} \frac{\lambda}{\Delta} \quad \text{for } -\frac{M}{2} \leq m \leq \frac{M}{2} - 1, \quad (13)$$

which cover a full period, there follows, for  $\left|\frac{m}{M} \frac{\lambda}{\Delta}\right| \leq 1$ ,

$$\bar{N}_f\left(\cos\left(\frac{m}{M} \frac{\lambda}{\Delta}\right)\right) \approx \frac{\Delta}{\lambda} \sum_{n=-\infty}^{+\infty} \exp(-i2\pi mn/M) G_f(\Delta n). \quad (14a)$$

The sum on the right-hand side can be accomplished via an  $M$ -point fast Fourier transform when collapsing is employed [5; p.5]. The resultant angles at which  $\bar{N}_f(\theta)$  is available are

$$\theta_m = \cos\left(\frac{m}{M} \frac{\lambda}{\Delta}\right), \quad \text{or} \quad \cos\theta_m = \frac{m}{M} \frac{\lambda}{\Delta} \quad \text{for } -\frac{M}{2} \leq m \leq \frac{M}{2} - 1, \quad (14b)$$

provided that  $\frac{|m|}{M} \frac{\lambda}{\Delta} \leq 1$ . These values are equally spaced in  $\cos\theta$  space.

The right-hand side of (12) can be rewritten in the form [5; pp. 3-4]

$$\begin{aligned} & \frac{1}{\lambda} \int_{-\infty}^{+\infty} dz \exp\left(-i\frac{2\pi}{\lambda} u z\right) G_f(z) \Delta \sum_{n=-\infty}^{+\infty} \delta(z - \Delta n) = \\ &= \bar{N}_f(\cos(u)) * \sum_{n=-\infty}^{+\infty} \delta\left(u - \frac{n\lambda}{\Delta}\right) = \sum_{n=-\infty}^{+\infty} \bar{N}_f\left(\cos\left(u - \frac{n\lambda}{\Delta}\right)\right), \quad (15a) \end{aligned}$$

where  $*$  denotes convolution. The separation of these aliased lobes (for  $n \neq 0$ ) is  $\lambda/\Delta$  on the  $u$  scale; then, since the extent of  $\bar{N}_f(\cos(u))$  is 2 on the  $u$  scale, overlapped aliasing lobes do not occur if  $\Delta < \lambda/2$ . This is a mathematical back-up to the claim under (12).

## DISCRETE FINITE-LENGTH ARRAY

The effect of a finite-length array can easily be incorporated by modifying (15a), so as to include weighting  $w(z)$ . Then, we have, for the estimated noise field directionality,

$$\begin{aligned} \frac{1}{\lambda} \int_{-\infty}^{+\infty} dz \exp\left(-i\frac{2\pi}{\lambda} u z\right) G_f(z) \Delta \sum_{n=-\infty}^{+\infty} \delta(z - \Delta n) w(z) &= \\ = \bar{N}_f(\cos(u)) \oplus \sum_{n=-\infty}^{+\infty} w\left(u - \frac{n\lambda}{\Delta}\right), \end{aligned} \quad (15b)$$

where window

$$w(u) = \frac{1}{\lambda} \int_{-\infty}^{+\infty} dz \exp\left(-i\frac{2\pi}{\lambda} u z\right) w(z). \quad (15c)$$

Thus, not only is the noise field directionality aliased at separations  $\lambda/\Delta$  in  $u$ , but, in addition, it is smoothed by window  $w$ . Sampling, per se, does not distort the estimated directionality, if done finely enough, that is,  $\Delta < \lambda/2$ . However, the finite length of the array always causes smearing, with a window width of the order of  $\lambda/L_z$ , where  $L_z$  is the effective length of weighting  $w(z)$ .

## PLANAR ARRAY

It is now most advantageous mathematically to locate the planar array in the  $x, y$  plane, that is, at  $z = 0$ . Then the exponential in (6) is independent of  $\cos\theta$ , and (6) reduces to

$$\begin{aligned} G_f(x, y) &= \int_0^{\pi} d\theta \int_{-\pi}^{\pi} d\phi \sin\theta N_f(\theta, \phi) \exp\left(i\frac{2\pi}{\lambda} \sin\theta (x \cos\phi + y \sin\phi)\right) = \\ &= \int_0^{\pi/2} d\theta \int_{-\pi}^{\pi} d\phi \sin\theta \tilde{N}_f(\theta, \phi) \exp\left(i\frac{2\pi}{\lambda} \sin\theta (x \cos\phi + y \sin\phi)\right), \quad (16) \end{aligned}$$

where

$$\tilde{N}_f(\theta, \phi) = N_f(\theta, \phi) + N_f(\pi - \theta, \phi)$$

$$\text{for } 0 \leq \theta \leq \pi/2, -\pi < \phi \leq \pi. \quad (17)$$

$\tilde{N}_f$  is the sum of the elemental components in symmetrically-arriving rays on opposite sides of the planar array; recall that  $\theta = \pi/2$  now corresponds to the plane of the array. Spatial correlation  $G_f(x, y)$  is the only function that can be measured (or estimated) from the planar array, and  $\tilde{N}_f(\theta, \phi)$  is the only field directionality function that can be determined. There is no possibility of undoing the summation of (17); this is a mathematical representation of the inherent two-sided symmetric response of a planar array. It is also one reason for choosing the planar array to lie along the  $\theta = \pi/2$  plane, since the totality of the two-dimensional field information is conveniently collapsed into a one-sided function of  $\theta$ , that is,  $0 \leq \theta \leq \pi/2$ .

## SOLUTION OF INTEGRAL EQUATION

Consider the two-dimensional Fourier transform of (16),

$$\begin{aligned} I(u, v) &\equiv \iint_{-\infty}^{+\infty} dx dy \exp\left(-i\frac{2\pi}{\lambda}(ux + vy)\right) G_f(x, y) = \\ &= \int_0^{\pi/2} d\theta \int_{-\pi}^{\pi} d\phi \sin\theta \tilde{N}_f(\theta, \phi) \lambda^2 \delta(u - \sin\theta \cos\phi) \delta(v - \sin\theta \sin\phi). \end{aligned} \quad (18)$$

Let

$$\alpha = \sin\theta \cos\phi, \beta = \sin\theta \sin\phi \quad \text{for } 0 \leq \theta \leq \pi/2, -\pi < \phi \leq \pi. \quad (19)$$

These relations can be inverted by using

$$\alpha + i\beta = \sin\theta \exp(i\phi), \quad (20)$$

to give

$$\sin\theta = |\alpha + i\beta| = (\alpha^2 + \beta^2)^{\frac{1}{2}}, \quad \phi = \arg(\alpha + i\beta). \quad (21)$$

Thus, (19) is a one-to-one two-dimensional transformation in the ranges  $0 \leq \theta \leq \pi/2$ ,  $-\pi < \phi \leq \pi$  allowed in (18). From (19) and (21), the Jacobian is

$$\begin{aligned} \frac{\partial(\alpha, \beta)}{\partial(\theta, \phi)} &= \begin{vmatrix} \cos\theta \cos\phi & -\sin\theta \sin\phi \\ \cos\theta \sin\phi & \sin\theta \cos\phi \end{vmatrix} = \\ &= \sin\theta \cos\theta = (\alpha^2 + \beta^2)^{\frac{1}{2}} (1 - \alpha^2 - \beta^2)^{\frac{1}{2}}. \end{aligned} \quad (22)$$

Substitution of these results in (18) yields

$$\begin{aligned}
 I(u, v) &= \lambda^2 \iint_{C_1} d\alpha d\beta F(\alpha, \beta) \left(1 - \alpha^2 - \beta^2\right)^{-\frac{1}{2}} \delta(u - \alpha) \delta(v - \beta) = \\
 &= \left\{ \begin{array}{ll} \lambda^2 F(u, v) \left(1 - u^2 - v^2\right)^{-\frac{1}{2}} & \text{for } u^2 + v^2 < 1 \\ 0 & \text{otherwise} \end{array} \right\}, \quad (23)
 \end{aligned}$$

where  $C_1$  is a circle of radius 1 located at the origin, and

$$F(\alpha, \beta) = \tilde{N}_f(\operatorname{asin}(|\alpha + i\beta|), \arg(\alpha + i\beta)). \quad (24)$$

Here,  $\operatorname{asin}$  is the principal value inverse sine function. From (23), (24), and (18), the noise field directionality is

$$\begin{aligned}
 \tilde{N}_f(\operatorname{asin}(|u + iv|), \arg(u + iv)) &= \\
 &= \frac{\left(1 - u^2 - v^2\right)^{\frac{1}{2}}}{\lambda^2} \iint_{-\infty}^{+\infty} dx dy \exp\left(-i\frac{2\pi}{\lambda} (ux + vy)\right) G_f(x, y) \\
 &\quad \text{for } u^2 + v^2 < 1. \quad (25)
 \end{aligned}$$

An alternative form is available by letting

$$u = \sin\theta \cos\phi, v = \sin\theta \sin\phi \quad \text{for } 0 \leq \theta \leq \pi/2, -\pi < \phi \leq \pi, \quad (26)$$

namely

$$\begin{aligned}
 \tilde{N}_f(\theta, \phi) &= \frac{\cos\theta}{\lambda^2} \iint_{-\infty}^{+\infty} dx dy \exp\left(-i\frac{2\pi}{\lambda} \sin\theta (x \cos\phi + y \sin\phi)\right) G_f(x, y) \\
 &\quad \text{for } 0 \leq \theta \leq \pi/2, -\pi < \phi \leq \pi. \quad (27)
 \end{aligned}$$

It is interesting to compare this form with starting result (16). An alternative, when the planar array lies in the  $y = 0$  plane, is given in appendix B.

## BEHAVIOR NEAR PLANE OF ARRAY

At first sight, the presence of the  $\cos\theta$  term in (27) would appear to be a problem for  $\theta = \pi/2$ , which is the plane of the array. However, the following example illustrates what is happening; let

$$\tilde{N}_f(\theta, \phi) = \delta(\theta - \theta_0) \delta(\phi - \phi_0) \quad \text{for } 0 < \theta_0 < \pi/2, -\pi < \phi_0 \leq \pi.$$

Then (16) yields spatial correlation

$$G_f(x, y) = \sin\theta_0 \exp\left(i\frac{2\pi}{\lambda} \sin\theta_0 (x \cos\phi_0 + y \sin\phi_0)\right).$$

The strength of this quantity tends to zero as  $\theta_0 \rightarrow 0$ . Substitution of this  $G_f(x, y)$  in (27) yields noise field directionality

$$\begin{aligned} \tilde{N}_f(\theta, \phi) &= \sin\theta_0 \cos\theta \delta(\sin\theta \cos\phi - \sin\theta_0 \cos\phi_0) \times \\ &\quad \times \delta(\sin\theta \sin\phi - \sin\theta_0 \sin\phi_0). \end{aligned}$$

By use of the property

$$\delta(ax + by) \delta(cx + dy) = \frac{\delta(x) \delta(y)}{|ad - bc|},$$

it may be shown that  $\tilde{N}_f(\theta, \phi)$  is  $\delta(\theta - \theta_0) \delta(\phi - \phi_0)$ , as expected; however, the trigonometric form shows  $\tilde{N}_f$  as the product of two terms, the first of which tends to 0 as  $\theta_0 \rightarrow 0$  or  $\pi/2$ , and the second of which has impulses with area which tends to infinity as  $\theta_0 \rightarrow 0$  or  $\pi/2$ . Thus, the  $\sin\theta_0$  and  $\cos\theta$  terms are not a problem since they are compensated by multiplicative terms; however, they may lead to inaccuracies in numerical computation.

## DISCRETE INFINITE-LENGTH ARRAY

The form in (25) gives the noise field directionality sum  $\tilde{N}_f$ , defined in (17), as a double Fourier transform of the two-dimensional spatial correlation function  $G_f(x, y)$ . If samples of  $G_f(x, y)$  at increments  $\Delta_x$  in  $x$  and  $\Delta_y$  in  $y$ , respectively, are available, an approximation to (25) is afforded, for  $u^2 + v^2 < 1$ , by

$$\begin{aligned} \tilde{N}_f(\arcsin(|u + iv|), \arg(u + iv)) \approx & \frac{(1 - u^2 - v^2)^{\frac{1}{2}}}{\lambda^2} \Delta_x \Delta_y \times \\ & \times \sum_{k=-\infty}^{+\infty} \sum_{j=-\infty}^{+\infty} \exp\left(-i\frac{2\pi}{\lambda}(u \Delta_x k + v \Delta_y j)\right) G_f(\Delta_x k, \Delta_y j). \end{aligned} \quad (28)$$

The summation on the right-hand side of (28) has periods  $\lambda/\Delta_x$  in  $u$ , and  $\lambda/\Delta_y$  in  $v$ . Since the sum  $\tilde{N}_f$  is defined within the circle  $u^2 + v^2 < 1$ , overlapped aliasing lobes will occur in (28) unless  $\Delta_x < \lambda/2$  and  $\Delta_y < \lambda/2$ ; that is, the spacings between samples of  $G_f(x, y)$  must be less than a half-wavelength at the temporal-frequency  $f$  of interest. We presume this to be true henceforth.

Now if we restrict  $u$  and  $v$  in (28) to the values

$$\begin{aligned} u_m &= \frac{m}{M} \frac{\lambda}{\Delta_x} \quad \text{for} \quad -\frac{M}{2} \leq m \leq \frac{M}{2} - 1, \\ v_n &= \frac{n}{N} \frac{\lambda}{\Delta_y} \quad \text{for} \quad -\frac{N}{2} \leq n \leq \frac{N}{2} - 1, \end{aligned} \quad (29)$$

both of which cover full periods in  $u$  and  $v$ , respectively, there follows

$$\tilde{N}_f(\arcsin(|u_m + iv_n|), \arg(u_m + iv_n)) \approx \frac{(1 - u_m^2 - v_n^2)^{\frac{1}{2}}}{\lambda^2} \times \\ \times \Delta_x \Delta_y \sum_{k=-\infty}^{+\infty} \sum_{j=-\infty}^{+\infty} \exp(-i2\pi mk/M - i2\pi nj/N) G_f(\Delta_x k, \Delta_y j), \quad (30)$$

provided that

$$|u_m + iv_n| = \left| \frac{m}{M} \frac{\lambda}{\Delta_x} + i \frac{n}{N} \frac{\lambda}{\Delta_y} \right| \leq 1. \quad (31)$$

The double sum in (30) can be accomplished as an  $M \times N$  two-dimensional fast Fourier transform, when collapsing is employed [5; p.5]. The resultant angles at which noise field directional-ity  $\tilde{N}_f(\theta, \phi)$  is available are

$$0 \leq \theta_{mn} = \arcsin \left| \frac{m}{M} \frac{\lambda}{\Delta_x} + i \frac{n}{N} \frac{\lambda}{\Delta_y} \right| \leq \frac{\pi}{2},$$

$$-\pi < \phi_{mn} = \arg \left( \frac{m}{M} \frac{\lambda}{\Delta_x} + i \frac{n}{N} \frac{\lambda}{\Delta_y} \right) \leq \pi, \quad (32)$$

or

$$\sin \theta_{mn} = \left| \frac{m}{M} \frac{\lambda}{\Delta_x} + i \frac{n}{N} \frac{\lambda}{\Delta_y} \right| = \\ = \left[ \left( \frac{m}{M} \frac{\lambda}{\Delta_x} \right)^2 + \left( \frac{n}{N} \frac{\lambda}{\Delta_y} \right)^2 \right]^{\frac{1}{2}}, \quad (33a)$$

where

$$-\frac{M}{2} \leq m \leq \frac{M}{2} - 1, \quad -\frac{N}{2} \leq n \leq \frac{N}{2} - 1, \quad (33b)$$

but remembering that (31) must remain true.

The right-hand side of (28) can be re-written in the form  
[5; pp. 3-4]

$$\begin{aligned}
& \frac{(1 - u^2 - v^2)^{\frac{1}{2}}}{\lambda^2} \iint_{-\infty}^{+\infty} dx dy \exp\left(-i\frac{2\pi}{\lambda}(ux + vy)\right) G_f(x, y) \times \\
& \times \Delta_x \sum_{k=-\infty}^{+\infty} \delta(x - \Delta_x k) \Delta_y \sum_{j=-\infty}^{+\infty} \delta(y - \Delta_y j) = \\
& = \tilde{N}_f(\text{asin}(|u + iv|), \arg(u + iv)) \oplus \\
& \oplus \sum_{k=-\infty}^{+\infty} \delta\left(u - \frac{k\lambda}{\Delta_x}\right) \oplus \sum_{j=-\infty}^{+\infty} \delta\left(v - \frac{j\lambda}{\Delta_y}\right). \quad (34a)
\end{aligned}$$

The separations of the aliased lobes (for  $(k, j) \neq (0, 0)$ ) are  $\lambda/\Delta_x$  on the  $u$  scale and  $\lambda/\Delta_y$  on the  $v$  scale. Then, since the extent of the noise field directionality  $\tilde{N}_f$  is  $u^2 + v^2 < 1$ , overlapped aliasing lobes do not occur if  $\Delta_x < \lambda/2$  and  $\Delta_y < \lambda/2$ . This is a quantitative restatement of the claims made in the sequel to (28).

## DISCRETE FINITE-LENGTH ARRAY

The effect of finite lengths in the x and y directions can be incorporated by modifying (34a) so as to include weighting  $w(x,y)$ . Then, we have, for the estimated noise field directionality,

$$\begin{aligned}
 & \frac{(1 - u^2 - v^2)^{\frac{1}{2}}}{\lambda^2} \iint_{-\infty}^{+\infty} dx dy \exp\left(-i\frac{2\pi}{\lambda}(ux + vy)\right) G_f(x,y) \times \\
 & \times \Delta_x \sum_{k=-\infty}^{+\infty} \delta(x - \Delta_x k) \Delta_y \sum_{j=-\infty}^{+\infty} \delta(y - \Delta_y j) w(x,y) = \\
 & = \tilde{N}_f(\text{asin}(|u + iv|), \arg(u + iv)) \oplus \sum_{k=-\infty}^{uv} \sum_{j=-\infty}^{+\infty} w\left(u - \frac{k\lambda}{\Delta_x}, v - \frac{j\lambda}{\Delta_y}\right)
 \end{aligned} \tag{34b}$$

where window

$$w(u,v) = \frac{1}{\lambda^2} \iint_{-\infty}^{+\infty} dx dy \exp\left(-i\frac{2\pi}{\lambda}(ux + vy)\right) w(x,y). \tag{34c}$$

Thus, not only is the noise field directionality aliased at separations  $\lambda/\Delta_x$  in u and  $\lambda/\Delta_y$  in v, but, in addition, it is smoothed by window W. Sampling alone does not distort the estimated directionality if done with  $\Delta_x < \lambda/2$  and  $\Delta_y < \lambda/2$ ; see (34a). However, the finite lengths of the array always smears, with window widths of the order of  $\lambda/L_x$  in u and  $\lambda/L_y$  in v, where  $L_x$  and  $L_y$  are the effective lengths of weighting  $w(x,y)$  in x and y, respectively.

## VOLUMETRIC ARRAY

We now have the full version (6):

$$G_f(x, y, z) = \int_0^{\pi} d\theta \int_{-\pi}^{\pi} d\phi \sin\theta N_f(\theta, \phi) \times \\ \times \exp\left(i \frac{2\pi}{\lambda} (x \sin\theta \cos\phi + y \sin\theta \sin\phi + z \cos\theta)\right). \quad (35)$$

However, since noise field directionality  $N_f$  is a function of two variables, while spatial correlation  $G_f$  has three arguments, some of the information in  $G_f$  is superfluous and must be reduced or collapsed in some fashion.

## SOLUTION OF INTEGRAL EQUATION

We begin by defining triple Fourier transform

$$I(u, v, w) = \iiint_{-\infty}^{+\infty} dx dy dz q(z) \exp\left(-i \frac{2\pi}{\lambda} (ux + vy + wz)\right) G_f(x, y, z) = \\ = \int_0^{\pi} d\theta \int_{-\pi}^{\pi} d\phi \sin\theta N_f(\theta, \phi) \lambda^2 \delta(u - \sin\theta \cos\phi) \delta(v - \sin\theta \sin\phi) \times \\ \times Q(w - \cos\theta), \quad (36)$$

where we use a weighting  $q(z)$  on the  $z$  variable, and define

$$Q(t) = \int_{-\infty}^{+\infty} dz \exp(-i2\pi t z / \lambda) q(z). \quad (37)$$

(If  $q(z) = 1$  for all  $z$ , then  $Q(t) = \lambda \delta(t)$ .)

We now break the right-hand side of (36) into two parts according to

$$\int_0^\pi d\theta \int_{-\pi}^\pi d\phi = \int_0^{\pi/2} d\theta \int_{-\pi}^\pi d\phi + \int_{\pi/2}^\pi d\theta \int_{-\pi}^\pi d\phi , \quad (38)$$

and in each region, we make the change of variable used in (19) et seq., namely

$$\alpha = \sin\theta \cos\phi, \quad \beta = \sin\theta \sin\phi. \quad (39)$$

Then  $\phi = \arg(\alpha + i\beta)$ , while

$$\theta = \begin{cases} \arcsin(|\alpha + i\beta|) & \text{for } 0 \leq \theta \leq \pi/2 \\ \pi - \arcsin(|\alpha + i\beta|) & \text{for } \pi/2 \leq \theta \leq \pi \end{cases} \quad (40)$$

and

$$\cos\theta = \begin{cases} (1 - \alpha^2 - \beta^2)^{\frac{1}{2}} & \text{for } 0 \leq \theta \leq \pi/2 \\ -(1 - \alpha^2 - \beta^2)^{\frac{1}{2}} & \text{for } \pi/2 \leq \theta \leq \pi \end{cases}. \quad (41)$$

Define, for future use,

$$s(\alpha, \beta) = (1 - \alpha^2 - \beta^2)^{\frac{1}{2}} \quad \text{for } \alpha^2 + \beta^2 \leq 1. \quad (42)$$

Using these results in (36), there follows

$$\begin{aligned} I(u, v, w) &= \lambda^2 \iint_{C_1} d\alpha d\beta s(\alpha, \beta)^{-1} F_1(\alpha, \beta) \delta(u-\alpha) \delta(v-\beta) Q(w-s(\alpha, \beta)) \\ &+ \lambda^2 \iint_{C_1} d\alpha d\beta s(\alpha, \beta)^{-1} F_2(\alpha, \beta) \delta(u-\alpha) \delta(v-\beta) Q(w+s(\alpha, \beta)), \end{aligned} \quad (43)$$

where  $C_1$  is a circle of radius 1 located at the origin, and

$$\left. \begin{aligned} F_1(\alpha, \beta) &= N_f(\arcsin(|\alpha + i\beta|), \arg(\alpha + i\beta)) \\ F_2(\alpha, \beta) &= N_f(\pi - \arcsin(|\alpha + i\beta|), \arg(\alpha + i\beta)) \end{aligned} \right\} \text{ for } |\alpha + i\beta| \leq 1. \quad (44)$$

Evaluating the integrals in (43), we have

$$\begin{aligned} I(u, v, w) = \lambda^2 s(u, v)^{-1} & \left[ F_1(u, v) Q(w - s(u, v)) + \right. \\ & \left. + F_2(u, v) Q(w + s(u, v)) \right] \quad \text{for } u^2 + v^2 < 1. \end{aligned} \quad (45)$$

### SIMULTANEOUS EQUATIONS

If we evaluate the triple Fourier transform in (36) at two different values of  $w$ , we have

$$\frac{s(u, v)}{\lambda^2} I(u, v, w_1) = F_1(u, v) Q(w_1 - s(u, v)) + F_2(u, v) Q(w_1 + s(u, v)) \quad (46)$$

and

$$\frac{s(u, v)}{\lambda^2} I(u, v, w_2) = F_1(u, v) Q(w_2 - s(u, v)) + F_2(u, v) Q(w_2 + s(u, v)). \quad (47)$$

Also, if we define

$$\begin{aligned} Q_n(+) &= Q(w_n + s(u, v)), \\ Q_n(-) &= Q(w_n - s(u, v)), \end{aligned} \quad (48)$$

and denominator

$$D = Q_1(-) Q_2(+) - Q_1(+) Q_2(-), \quad (49)$$

the solutions to (46) and (47) are

$$\left. \begin{aligned} F_1(u, v) &= \frac{s(u, v)}{\lambda^2 D} \left( Q_2(+) I(u, v, w_1) - Q_1(+) I(u, v, w_2) \right) \\ F_2(u, v) &= \frac{s(u, v)}{\lambda^2 D} \left( Q_1(-) I(u, v, w_2) - Q_2(-) I(u, v, w_1) \right) \end{aligned} \right\} \begin{array}{l} \text{for} \\ u^2 + v^2 < 1, \end{array} \quad (50)$$

provided that  $D \neq 0$ . Function  $s$  is defined in (42).

There is a great deal of leeway in these solutions. Namely,  $Q(t)$  in (37) is arbitrary, and the values  $w_1$  and  $w_2$  are arbitrary as well; the only restriction is that  $D$  in (49) not be zero. In the special case where weighting  $q(z)$  in (36) is real and even, then  $Q(t)$  in (37) is also real and even; we presume this to be the case henceforth. If we then choose  $w_2 = -w_1$ , (49) becomes

$$D = Q^2(w_1 - s(u, v)) - Q^2(w_1 + s(u, v)). \quad (51)$$

If the effective length of weighting  $q(z)$  is  $L_z$ , a representative plot of  $Q^2(t)$  is displayed in figure 2. For small  $s$ , a good location for  $w_1$  is at the point where  $Q^2(t)$  has its maximum slope. For larger  $s$ , a value for  $w_1$  near  $s$  would guarantee a large value for  $D$  in (51).

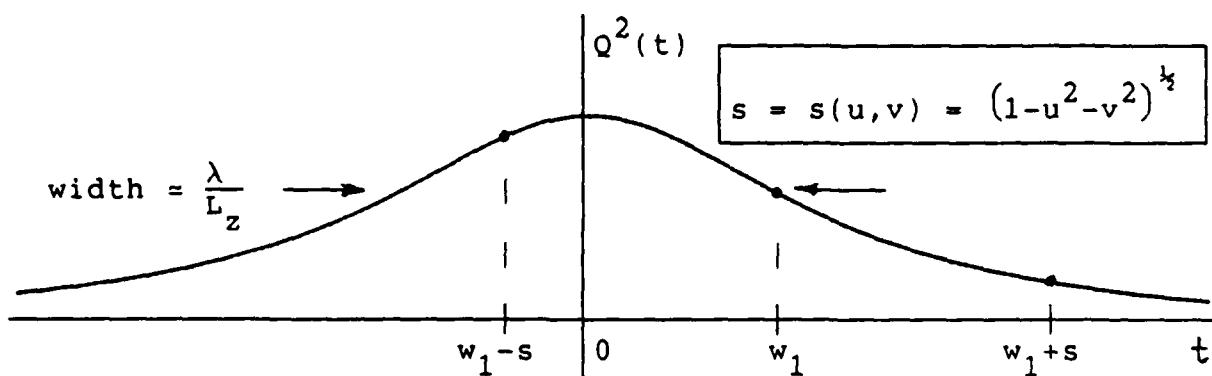


Figure 2. Window Function  $Q^2$

## ANGULAR REPRESENTATIONS

If we make the substitutions

$$u = \sin\theta \cos\phi, v = \sin\theta \sin\phi \quad \text{for } 0 \leq \theta \leq \pi/2, -\pi < \phi \leq \pi, \quad (52)$$

then (42), (44), and (51) yield

$$\left. \begin{aligned} s(\sin\theta \cos\phi, \sin\theta \sin\phi) &= \cos\theta \\ F_1(\sin\theta \cos\phi, \sin\theta \sin\phi) &= N_f(\theta, \phi) \\ F_2(\sin\theta \cos\phi, \sin\theta \sin\phi) &= N_f(\pi - \theta, \phi) \\ D &= Q^2(w_1 - \cos\theta) - Q^2(w_1 + \cos\theta) \end{aligned} \right\} \begin{aligned} &\text{for} \\ &0 \leq \theta \leq \pi/2, -\pi < \phi \leq \pi. \end{aligned} \quad (53)$$

Then (50) becomes

$$N_f(\theta, \phi) = \frac{\cos\theta}{\lambda^2} \frac{Q(w_1 - \cos\theta) I(+) - Q(w_1 + \cos\theta) I(-)}{Q^2(w_1 - \cos\theta) - Q^2(w_1 + \cos\theta)}, \quad (54a)$$

$$N_f(\pi - \theta, \phi) = \frac{\cos\theta}{\lambda^2} \frac{Q(w_1 - \cos\theta) I(-) - Q(w_1 + \cos\theta) I(+)}{Q^2(w_1 - \cos\theta) - Q^2(w_1 + \cos\theta)}, \quad (54b)$$

for  $0 \leq \theta \leq \pi/2, -\pi < \phi \leq \pi,$

where we define

$$\begin{aligned} I(\pm) &= I(\sin\theta \cos\phi, \sin\theta \sin\phi, \pm w_1) = \iiint_{-\infty}^{+\infty} dx dy dz q(z) \times \\ &\times \exp\left(-i\frac{2\pi}{\lambda} (x \sin\theta \cos\phi + y \sin\theta \sin\phi \pm z w_1)\right) G_f(x, y, z), \quad (55) \end{aligned}$$

upon use of (36). We repeat that these results for the noise field directionality apply only for  $Q(t)$  real and even; otherwise,  $Q(t)$  and  $w_1$  are arbitrary.

An alternative form to (54b) is available, if desired, by the substitution  $\theta' = \pi - \theta$ , namely

$$N_f(\theta', \phi) = \frac{\cos\theta'}{\lambda^2} \frac{Q(w_1 + \cos\theta') I'(-) - Q(w_1 - \cos\theta') I'(+) }{Q^2(w_1 - \cos\theta') - Q^2(w_1 + \cos\theta')}$$

for  $\pi/2 \leq \theta' \leq \pi$ ,  $-\pi < \phi \leq \pi$ , (56)

where

$$I'(\pm) = I(\sin\theta' \cos\phi, \sin\theta' \sin\phi, \pm w_1). \quad (57)$$

The most extensive calculation required here is that given by (55); rewriting it differently,

$$\begin{aligned} I(u, v, \pm w_1) &= \iint_{-\infty}^{+\infty} dx dy \exp\left(-i\frac{2\pi}{\lambda}(ux + vy)\right) \times \\ &\times \int_{-\infty}^{+\infty} dz \exp\left(-i\frac{2\pi}{\lambda}(\pm w_1)z\right) q(z) G_f(x, y, z). \end{aligned} \quad (58)$$

The innermost integral, the Fourier transform on  $z$ , only needs to be accomplished for the two values  $\pm w_1$ , whereas the outer integrals must be done for ranges of  $u$  and  $v$ . This is the collapsing operation alluded to under (35). On the other hand, the inner integral must be repeated for every  $x, y$  value of interest;

nevertheless, (58) is not as difficult as a three-dimensional Fourier transform.

An example of this procedure for the volumetric array is carried out in appendix C; it illustrates the care that must be taken with respect to the  $\theta$  variable in (54).

## SUMMARY

The noise field directionality for the cases of one-, two-, and three-dimensional arrays have been solved for, explicitly, in terms of the appropriate spatial correlation available in each case. In the one-dimensional case, only the polar directionality can be determined, while in the two-dimensional case, the sum of symmetrically arriving rays on both sides of the planar array can be evaluated. For the three-dimensional case, all ambiguity can be eliminated, but the overdetermined nature of the problem requires some collapsing of information and leaves many options to consider. For example, one could let the volumetric array be a thin-shelled sphere; however, the resulting two-dimensional integral equation for the noise field directionality cannot be solved explicitly. The attractive feature of large stacked planar arrays is that it permits the use of Fourier transforms and, therefore, an explicit expression for the noise field directionality in terms of the three-dimensional spatial correlation. Also, Fourier transforms are efficiently evaluated by the use of fast Fourier transforms.

In this investigation, we have presumed exact knowledge of the spatial correlation  $G_f(z)$  or  $G_f(x,y)$  or  $G_f(x,y,z)$ , depending on the dimensionality of the array employed. In practice,  $G_f$  must be estimated from measurements made from a physical array; in this case, maximum advantage should be taken of the stationarity and homogeneity of the noise field. Thus, for a line array of equi-spaced elements,  $G_f(n\Delta)$  should be estimated from all the

available pairs of elements that have separation  $n\Delta$  in space and over the total available observation time that data have been recorded on all elements.

A comparison [6] is underway between the methods of this report and the Fourier series method given in [4], at least for the line array. Results are similar, but not identical; in particular, the aliasing of the Fourier series method is more severe than for the Fourier integral approach.

## APPENDIX A. ALTERNATIVE LOCATION OF LINEAR ARRAY

If we locate the line array along the x axis, that is,  
 $y = z = 0$ , then (6) reduces to

$$\begin{aligned} G_f(x) &= \int_0^{\pi} d\theta \int_{-\pi}^{\pi} d\phi \sin\theta N_f(\theta, \phi) \exp\left(i\frac{2\pi}{\lambda} x \sin\theta \cos\phi\right) = \\ &= \int_0^{\pi/2} d\theta \int_0^{\pi} d\phi \sin\theta \tilde{N}_f(\theta, \phi) \exp\left(i\frac{2\pi}{\lambda} x \sin\theta \cos\phi\right), \quad (A-1) \end{aligned}$$

where

$$\tilde{N}_f(\theta, \phi) = N_f(\theta, \phi) + N_f(\pi - \theta, \phi) + N_f(\theta, -\phi) + N_f(\pi - \theta, -\phi) \quad (A-2)$$

for  $0 \leq \theta \leq \pi/2$ ,  $0 \leq \phi \leq \pi$ . Therefore, Fourier transform

$$\begin{aligned} I(u) &\equiv \int_{-\infty}^{+\infty} dx \exp\left(-i\frac{2\pi}{\lambda} u x\right) G_f(x) = \\ &= \int_0^{\pi/2} d\theta \int_0^{\pi} d\phi \sin\theta \tilde{N}_f(\theta, \phi) \lambda \delta(u - \sin\theta \cos\phi). \quad (A-3) \end{aligned}$$

Now let

$$s = \sin\theta, \quad t = \cos\phi, \quad (A-4)$$

which are one-to-one transformations in the ranges allowed in integral (A-3). Then

$$I(u) = \lambda \int_0^1 \frac{ds}{(1 - s^2)^{\frac{1}{2}}} \int_{-1}^1 \frac{dt}{(1 - t^2)^{\frac{1}{2}}} N_f(\sin(s), \cos(t)) s \delta(u - ts). \quad (A-5)$$

The innermost integral on  $t$  yields

$$\left\{ \begin{array}{ll} \frac{N_f(\sin(s), \cos(u/s))}{(1 - u^2/s^2)^{\frac{1}{2}}} & \text{for } |u| < s \\ 0 & \text{for } |u| > s \end{array} \right\}, \quad (A-6)$$

thereby giving

$$I(u) = \lambda \int_{|u|}^1 \frac{ds}{(1 - s^2)^{\frac{1}{2}}} \frac{s}{(s^2 - u^2)^{\frac{1}{2}}} N_f(\sin(s), \cos(u/s)) \quad \text{for } |u| < 1. \quad (A-7)$$

This integral equation for noise field directionality  $N_f$  is more general than Abel's integral equation, because limit  $u$  is also involved in one of the arguments of  $N_f$ . We have been unable to simplify (A-7) and extract any simple descriptor of the noise field directionality analogous to (8). Placing the linear array along the  $y$  axis, instead, encounters the same problem.

## APPENDIX B. ALTERNATIVE LOCATION OF PLANAR ARRAY

Suppose the planar array lies in the  $x, z$  plane, that is,  
 $y = 0$ . Then (6) becomes

$$\begin{aligned} G_f(x, z) &= \int_0^{\pi} d\theta \int_{-\pi}^{\pi} d\phi \sin\theta N_f(\theta, \phi) \exp\left(i\frac{2\pi}{\lambda}(x \sin\theta \cos\phi + z \cos\theta)\right) = \\ &= \int_0^{\pi} d\theta \int_0^{\pi} d\phi \sin\theta \underline{N}_f(\theta, \phi) \exp\left(i\frac{2\pi}{\lambda}(x \sin\theta \cos\phi + z \cos\theta)\right), \quad (B-1) \end{aligned}$$

where

$$\underline{N}_f(\theta, \phi) = N_f(\theta, \phi) + N_f(\theta, -\phi) \quad \text{for } 0 \leq \theta \leq \pi, 0 \leq \phi \leq \pi. \quad (B-2)$$

Then

$$\begin{aligned} I(u, v) &\equiv \iint_{-\infty}^{+\infty} dx dz \exp\left(-i\frac{2\pi}{\lambda}(ux + vz)\right) G_f(x, z) = \\ &= \int_0^{\pi} d\theta \int_0^{\pi} d\phi \sin\theta \underline{N}_f(\theta, \phi) \lambda^2 \delta(u - \sin\theta \cos\phi) \delta(v - \cos\theta). \quad (B-3) \end{aligned}$$

Now let

$$s = \sin\theta \cos\phi, \quad t = \cos\theta, \quad (B-4)$$

for which the Jacobian is

$$\frac{\partial(s, t)}{\partial(\theta, \phi)} = \sin^2\theta \sin\phi = (1 - t^2)^{\frac{1}{2}} (1 - s^2 - t^2)^{\frac{1}{2}}. \quad (B-5)$$

Then

$$\begin{aligned}
 I(u, v) &= \lambda^2 \iint_{C_1} ds dt \frac{\underline{N}_f\left(\cos(t), \cos\left(s(1-t^2)^{-\frac{1}{2}}\right)\right)}{(1-s^2-t^2)^{\frac{1}{2}}} \delta(u-s) \delta(v-t) = \\
 &= \left\{ \begin{array}{ll} \frac{\lambda^2}{(1-u^2-v^2)^{\frac{1}{2}}} \underline{N}_f\left(\cos(v), \cos\left(\frac{u}{(1-v^2)^{\frac{1}{2}}}\right)\right) & \text{for } u^2 + v^2 < 1 \\ 0 & \text{otherwise} \end{array} \right\}. \\
 &\quad (B-6)
 \end{aligned}$$

Thus, we have the explicit representation for the noise field directionality,

$$\begin{aligned}
 \underline{N}_f\left(\cos(v), \cos\left(\frac{u}{(1-v^2)^{\frac{1}{2}}}\right)\right) &= \frac{(1-u^2-v^2)^{\frac{1}{2}}}{\lambda^2} \iint_{-\infty}^{+\infty} dx dz \times \\
 &\quad \times \exp\left(-i\frac{2\pi}{\lambda}(ux + vz)\right) G_f(x, z) \quad \text{for } u^2 + v^2 < 1. \quad (B-7)
 \end{aligned}$$

If we now let

$$u = \sin\theta \cos\phi, \quad v = \cos\theta, \quad (B-8)$$

this becomes

$$\begin{aligned}
 \underline{N}_f(\theta, \phi) &= \frac{\sin\theta \sin\phi}{\lambda^2} \iint_{-\infty}^{+\infty} dx dz \exp\left(-i\frac{2\pi}{\lambda}(x \sin\theta \cos\phi + z \cos\theta)\right) \times \\
 &\quad \times G_f(x, z) \quad \text{for } 0 \leq \theta \leq \pi, 0 \leq \phi \leq \pi. \quad (B-9)
 \end{aligned}$$

This is a viable alternative to (27). Compare with starting result (B-1).

## APPENDIX C. EXAMPLE FOR VOLUMETRIC ARRAY

Let the noise field directionality be given by

$$N_f(\theta, \phi) = \delta(\theta - \theta_0) \delta(\phi - \phi_0), \quad 0 < \theta_0 < \pi, \quad -\pi < \phi_0 \leq \pi. \quad (C-1)$$

Notice that arrival angle  $\theta_0$  can range over an interval of length  $\pi$ . We distinguish two cases:

$$A: \quad 0 < \theta_0 < \pi/2,$$

$$B: \quad \pi/2 < \theta_0 < \pi. \quad (C-2)$$

From (35), the three-dimensional spatial correlation is

$$G_f(x, y, z) = \sin \theta_0 \exp \left( i \frac{2\pi}{\lambda} (x \sin \theta_0 \cos \phi_0 + y \sin \theta_0 \sin \phi_0 + z \cos \theta_0) \right). \quad (C-3)$$

The problem addressed here is the reestablishment of (C-1) by means of the solution procedure given in (54)-(57). Recall that  $Q(t)$  is real and even.

First, substituting (C-3) in (55), there follows

$$I(\pm) = \sin \theta_0 \lambda^2 \delta(\sin \theta \cos \phi - \sin \theta_0 \cos \phi_0) \times \\ \times \delta(\sin \theta \sin \phi - \sin \theta_0 \sin \phi_0) Q(\pm w_1 - \cos \theta_0). \quad (C-4)$$

Now, when we recall that  $\theta$  is limited to  $(0, \pi/2)$  in (54), the delta functions in (C-4) are located at

$$A: \quad \theta = \theta_0, \quad \phi = \phi_0,$$

$$\text{or} \quad B: \quad \theta = \pi - \theta_0, \quad \phi = \phi_0. \quad (C-5)$$

By means of the two-dimensional transformation employed in (19)-(22), we find that

$$A: I(\pm) = \frac{\lambda^2}{\cos\theta_0} \delta(\theta - \theta_0) \delta(\phi - \phi_0) Q(w_1 \mp \cos\theta_0) ,$$

$$B: I(\pm) = \frac{\lambda^2}{|\cos\theta_0|} \delta(\theta - \pi + \theta_0) \delta(\phi - \phi_0) Q(w_1 \mp \cos\theta_0) . \quad (C-6)$$

Substitution of (C-6) in the numerator of (54a) yields

$$A: \cos\theta \frac{\lambda^2}{\cos\theta_0} \delta(\theta - \theta_0) \delta(\phi - \phi_0) C(\theta, \theta_0) ,$$

$$B: \cos\theta \frac{\lambda^2}{|\cos\theta_0|} \delta(\theta - \pi + \theta_0) \delta(\phi - \phi_0) C(\theta, \theta_0) , \quad (C-7)$$

where

$$C(\theta, \theta_0) = Q(w_1 - \cos\theta) Q(w_1 - \cos\theta_0) - Q(w_1 + \cos\theta) Q(w_1 + \cos\theta_0) . \quad (C-8)$$

But since

$$C(\theta_0, \theta_0) = Q^2(w_1 - \cos\theta_0) - Q^2(w_1 + \cos\theta_0) ,$$

$$C(\pi - \theta_0, \theta_0) = 0 , \quad (C-9)$$

we find that

$$N_f(\theta, \phi) = \begin{cases} \delta(\theta - \theta_0) \delta(\phi - \phi_0) & \text{for case A} \\ 0 & \text{for case B} \end{cases} . \quad (C-10)$$

On the other hand, substitution of (C-6) in the numerator of (54b) yields

$$A: \cos\theta \frac{\lambda^2}{\cos\theta_0} \delta(\theta - \theta_0) \delta(\phi - \phi_0) D(\theta, \theta_0),$$

$$B: \cos\theta \frac{\lambda^2}{|\cos\theta_0|} \delta(\theta - \pi + \theta_0) \delta(\phi - \phi_0) D(\theta, \theta_0), \quad (C-11)$$

where

$$D(\theta, \theta_0) = Q(w_1 - \cos\theta) Q(w_1 + \cos\theta_0) - Q(w_1 + \cos\theta) Q(w_1 - \cos\theta_0). \quad (C-12)$$

But since

$$D(\theta_0, \theta_0) = 0,$$

$$D(\pi - \theta_0, \theta_0) = Q^2(w_1 + \cos\theta_0) - Q^2(w_1 - \cos\theta_0), \quad (C-13)$$

we find that

$$N_f(\pi - \theta, \phi) = \begin{cases} 0 & \text{for case A} \\ \delta(\theta - \pi + \theta_0) \delta(\phi - \phi_0) & \text{for case B} \end{cases}. \quad (C-14)$$

This last case could be written in a form similar to (56) as

$$N_f(\theta', \phi) = \delta(\theta' - \theta_0) \delta(\phi - \phi_0) \quad \text{for } \pi/2 < \theta' < \pi. \quad (C-15)$$

In any event, (C-10) and (C-14) confirm starting result (C-1) for the noise field directionality.

REFERENCES

1. A. H. Nuttall, **Estimation of Noise Directionality Spectrum**, NUSC Technical Memorandum TC-211-71, Naval Underwater Systems Center, New London, CT, 29 October 1971; also NUSC Technical Report 4345, 1 September 1972.
2. A. H. Nuttall, **Estimation of Noise Directionality Spectrum; Extensions and Generalizations**, NUSC Technical Memorandum TC-6-73, Naval Underwater Systems Center, New London, CT, 7 May 1973.
3. N. Yen, **Ambient Sea Noise Directionality: Measurement and Processing**, NUSC Technical Report 5545, Naval Underwater Systems Center, New London, CT, 28 February 1977.
4. J. H. Wilson, "Signal Detection and Localization Using the Fourier Series Method (FSM) and Cross-Sensor Data," **Journal of the Acoustical Society of America**, volume 73, number 5, pages 1648-1656, May 1983.
5. A. H. Nuttall, **Alias-Free Wigner Distribution Function and Complex Ambiguity Function for Discrete-Time Samples**, NUSC Technical Report 8533, Naval Underwater Systems Center, New London, CT, 14 April 1989.
6. A. H. Nuttall, **Estimation of Noise Field Directionality; Comparison with Fourier Series Method**, NUSC Technical Report 8599, Naval Underwater Systems Center, New London, CT, 15 October 1989.

A. H. Nuttall

INITIAL DISTRIBUTION LIST

Addressee	No. of Copies
ADMIRALTY RESEARCH ESTABLISHMENT, London, England (Dr. L. Lloyd)	1
ADMIRALTY UNDERWATER WEAPONS ESTABLISHMENT, Dorset, England	1
APPLIED PHYSICS LAB, JOHN HOPKINS (John C. Stapleton)	1
APPLIED PHYSICS LAB, U. WASHINGTON (C. Eggan)	1
APPLIED RESEARCH LAB, PENN STATE,(Frank W. Symons)	1
APPLIED RESEARCH LAB, U. TEXAS (Dr. M. Frazer)	1
APPLIED SEISMIC GROUP, Cambridge, MA (Richard Lacoss)	1
A & T, Stonington, Ct (H. Jarvis)	1
ASTRON RESEARCH & ENGINEERING, Santa Monica, CA (Dr. Allen Piersol)	1
BBN, Arlington, Va. (Dr. H. Cox)	1
BBN, Cambridge, MA (H. Gish)	1
BBN, New London, Ct. (Dr. P. Cable)	1
BELL COMMUNICATIONS RESEARCH, Morristown, NJ (J. Kaiser and D. Sunday (Library))	2
BENDAT, Julius Dr., Los Angeles, CA	1
BLEINSTEIN, Norman Dr., Denver, CO	1
BROWN UNIV, Providence, RI (Documents Library)	1
CANBERRA COLLEGE OF ADV. EDUC, BELCONNEN, A.C.T. Australia (P. Morgan)	1
COAST GUARD ACADEMY, New London, CT (Prof. J. Wolcin)	1
COAST GUARD R & D, Groton, CT (Library)	1
COGENT SYSTEMS, INC, (J. Costas)	1
COHEN, Leon Dr., Bronx, NY	1
CONCORDIA UNIVERSITY H-915-3, Montreal, Quebec Canada (Prof. Jeffrey Krolik)	1
CNO (NOP-098)	1
CNR-OCNR-00, 10, 12, 13, 20	5
DALHOUSIE UNIV., Halifax, Nova Scotia, Canada (Dr. B. Ruddick)	1
DAVID W. TAYLOR RESEARCH CNTR, Annapolis, MD (P. Prendergast, Code 2744)	1
DARPA, Arlington, VA (A. Ellinhorpe)	1
DEFENCE RESEARCH CENTER, Adelaide, Australia (Library)	1
DEFENCE RESEARCH ESTAB. ATLANTIC, Dartmouth, Nova Scotia (Library)	1
DEFENCE RESEARCH ESTAB. PACIFIC, Victoria, Canada (Dr. D. Thomson)	1
DEFENCE SCIENTIFIC ESTABLISHMENT, MINISTRY OF DEFENCE, Auckland, New Zealand (Dr. L. Hall)	1
DEFENSE SYSTEMS, INC, Mc Lean, VA (Dr. G. Sebestyen)	1
DIA	1
DIAGNOSTIC/RETRIEVAL SYSTEMS, INC, Tustin, CA. (J. Williams)	1
DTIC	1
DTRC	1
DREXEL UNIV, (Prof. S. Kesler)	1
EDO CORP, College Point, NY (M. Blanchard)	1

## INITIAL DISTRIBUTION LIST (Cont'd.)

Addressee	No. of Copies
EG&G, Manassas, VA (D. Frohman)	1
GENERAL ELECTRIC CO, Moorestown, NJ (Dr. Mark Allen 108-102)	1
GENERAL ELECTRIC CO., Philadelphia, PA (T. J. McFall)	1
GENERAL ELECTRIC CO, Pittsfield, MA (R. W. Race)	1
GENERAL ELECTRIC CO, Syracuse, NY ( J. L. Rogers, Dr. A. M. Vural and D. Winfield)	3
HAHN, Wm, Wash, DC	1
HARRIS SCIENTIFIC SERVICES, Dobbs Ferry, NY (B. Harris)	1
HARVARD UNIVERSITY, Gordon McKay Library	1
HONEYWELL ENGR SERV CNTR, Poulsbro, WA (C. Schmid)	1
HUGHES AIRCRAFT, Fullerton, CA (S. Autrey)	1
HUGHES AIRCRAFT, Buena Park, CA (T. Posch)	1
IBM, Manassas, VA (G. Demuth)	1
INDIAN INSTITUTE OF TECHNOLOGY, Madras, India (Dr. K. M. M. Prabhu)	1
INTERSTATE ELECTRONICS CORP, Anaheim, CA (R. Nielsen, 8011)	1
JOHNS HOPKINS UNIV, Laurel, MD (J. C. Stapleton)	1
KILDARE CORP, New London, CT (Dr. R. Mellen)	1
LINCOM CORP., Northboro, MA (Dr. T. Schonhoff)	1
MAGNAVOX ELEC SYSTEMS CO, Ft. Wayne, IN (R. Kenefic)	1
MALTZ, FRED, Sunnyvale, CA	1
MARINE BIOLOGICAL LAB, Woods Hole, MA	1
MARINE PHYSICAL LABORATORY SCRIPPS	1
MASS. INSTITUTE OF TECHNOLOGY (Prof. A. Baggaroer, Barker Engineering Library)	2
MBS SYSTEMS, Norwalk, CT (A. Winder)	1
MIDDLETON, DAVID, NY, NY	1
NADC (5041, M. Mele)	1
NAIR-03	1
NASH, Harold E., Quaker Hill, CT	1
NATIONAL RADIO ASTRONOMY OBSERVATORY, Charlottesville, VA (F. Schwab)	1
NATIONAL SECURITY AGENCY, FT. Meade, MD (Dr. James R. Maar, R51)	1
NATO SACLANT ASW RESEARCH CENTRE, APO NY, NY (Library, R. E. Sullivan and G. Tacconi)	3
NAVAL OCEAN SYSTEMS CENTER, San Diego, CA (J. M. Alsup, Code 635	1
NCSC	1
NEPRF	1
NORDA	1
NRS, Washington, DC (Dr. Philip B. Abraham, Code 5131)	1
NRL UND SOUND REF DET, Orlando, FL	1
NAVAL INTELLIGENCE COMMAND	1
NAVAL INTELLIGENCE SUPPORT CENTER	1
NAVAL OCEAN SYSTEMS CENTER, San Diego, CA (James M. Alsup, Code 635)	1
NAVAL OCEANOGRAPHY OFFICE	1
NAVAL SYSTEMS DIV., SIMRAD SUBSEA A/S, Norway (E. B. Lunde)	1

INITIAL DISTRIBUTION LIST (Cont'd.)

Addressee	No. of Copies
NICHOLS RESEARCH CORP., Wakefield, MA (T. Marzetta)	1
NORDA (Dr. B. Adams)	1
NORDA (Code 345) N STL Station, MS (R. Wagstaff)	1
NORTHEASTERN UNIV. Boston, MA (Prof. C. L. Nikias)	1
NORWEGIAN DEFENCE RESEARCH EST, Norway (Dr J. Glattetre)	1
NOSC, (James M. Alsup, Code 635, C. Sturdevant; 73, J. Lockwood, F. Harris, 743, R. Smith; 62, R. Thuleen)	6
NPROC	1
NPS, Monterey, CA (C. W. Therrien, Code 62 Ti)	2
NRL, Washington, DC (Dr. J. Buccaro, Dr. E. Franchi, Dr. P. Abraham, Code 5132, A. A. Gerlach, W. Gabriel (Code 5370), and N. Yen (Code 5130)	6
NRL, Arlington, VA (N. L. Gerr, Code 1111)	1
NSWC	1
NSWC DET Ft. Lauderdale	1
NSWC WHITE OAK LAB	1
NUSC DET TUDOR HILL	1
NUSC DET WEST PALM BEACH (Dr. R. Kennedy Code 3802)	1
NWC	1
ORI CO, INC, New London, CT (G. Assard)	1
ORINCON CORP., Columbia, MD (S. Larry Marple)	1
PAPOUTSANIS, P. D., Athens, Greece	1
PENN STATE UNIV., State College, PA (F. Symons)	1
PIERSOLL ENGR CO, Woodland Hills, CA (Dr. Allen G. Piersol)	1
POHLER, R., Austin, TX	1
POLETTI, Mark A., Acoustics Research Centre, School of Architecture, Univ. of Auckland, Auckland, New Zealand	1
PROMETHEUS, INC, Sharon, MA (Dr. J. Byrnes)	1
PROMETHEUS INC, Newport, RI (Michael J. Barrett)	1
PRICE, Robert Dr. Lexington, MA	1
PURDUE UNIV, West Lafayette, IN (N. Srinivasa)	1
RAISBECK, Dr. Gordon, Portland, ME	1
RAN RESEARCH LAB, Darlinghurst, Australia	1
RAYTHEON CO, Portsmouth, RI (J. Bartram, R. Connor) and S. S. Reese)	3
RICHTER, W., Annandale, VA.	1
ROCKWELL INTERNATIONAL CORP, Anaheim, CA (L. Einstein and Dr. D. Elliott)	2
ROYAL MILITARY COLLEGE OF CANADA, (Prof. Y. Chan)	1
RUTGERS UNIV., Piscataway, NJ (Prof. S. Orfanidis)	1
RCA CORP, Moorestown, NJ (H. Upkowitz)	1
SACLANT UNDERSEA RESEARCH CENTRE, APO NY NY (Dr. John Ianniello, Dr. S. Stergiopolous and Giorgio Tacconi, Library)	4
SAIC, Falls Church, VA (Dr. P. Mikhalevsky)	1
SAIC, New London, CT (Dr. F. Dinapoli)	1
SANDIA NATIONAL LABORATORY (J. Claasen)	1
SCHULKIN, Dr. Morris, Potomac, MD	1
SEA-00, 63, 63X	3
SIMON FRASER UNIV, British Columbia, Canada (Dr. Edgar Velez)	1

INITIAL DISTRIBUTION LIST (Cont'd.)

Addressee	No. of Copies
SONAR SURVEILLANCE GROUP, Darlinghurst, Australia	1
SOUTHEASTERN MASS. UNIV (Prof. C. H. Chen)	1
SPAWARS-00, 04, 005, PD-80 and PMW-181	5
SPERRY CORP, Great Neck, NY	1
STATE UNIV. OF NY AT STONY BROOK (Prof. M. Barkat)	1
TEL-AVIV UNIV, Tel-Aviv, Israel (Prof. E. Weinstein)	1
TOYON RESEARCH CORP, Goleta, CA (M. Van Blaricum)	1
TRACOR, INC, Austin, TX (Dr. T Leih and J. Wilkinson)	2
TRW FEDERAL SYSTEMS GROUP, Fairfax, VA (R. Prager)	1
UNITED ENGINEERING CENTER, Engr. Societies Library, NY, NY	1
UNIV. OF AUCKLAND, New Zealand (Dr. Murray D. Johns)	1
UNIV. OF ALBERTA, Edmonton, Alberta, CANADA (K. Yeung)	1
UNIV OF CA, San Diego, CA (Prof. C. Helstrom)	1
UNIV OF COLORADO, Boulder, CO (Prof. L. Scharf)	1
UNIV. OF CT, Storrs, CT. (Library and Prof. C. Knapp)	2
UNIV OF FLA, Gainesville, FL (D. Childers)	1
UNIV OF ILLINOIS, Urbana, IL 61801 (Dr. Douglas L. Jones)	1
UNIV OF MICHIGAN, Ann Arbor, MI (William J. Williams)	1
UNIV. OF MINN, Minneapolis, Mn (Prof. M. Kaveh)	1
UNIV. OF NEWCASTLE, Newcastle, NSW, Canada (Prof. A. Cantoni)	1
UNIV. OF QUEENSLAND, St. Lucia, Queensland 4067, Australia (Dr. Boualem Boashash)	1
UNIV. OF RI, Kingston, RI (Prof. G. F. Boudreaux-Bartels, Library, Prof. S. Kay, and Prof. D. Tufts)	4
UNIV. OF ROCHESTER, Rochester, NY (Prof. E. Titlebaum)	1
UNIV. OF SOUTHERN CA., LA. (Prof. William C. Lindsey, Dr. Andreas Polydoros, PHE 414)	2
UNIV. OF STRATHCLYDE, ROYAL COLLEGE, Glasgow, Scotland (Prof. T. Durrani)	1
UNIV. OF TECHNOLOGY, Loughborough, Leicestershire, England (Prof. J. Griffiths)	1
UNIV. OF WASHINGTON, Seattle (Prof. D. Lytle)	1
URICK, ROBERT, Silver Springs, MD	1
US AIR FORCE, Maxwell AF Base, AL (Library)	1
VAN ASSELT, Henrik, USEA S.P.A., La Spezia, Italy	1
VILLANOVA UNIV, Villanova, PA (Prof. Moeness G. Amin)	1
WEAPONS SYSTEMS RESEARCH LAB, Adelaide, Australia	2
WERBNER, A., Medford, MA	1
WESTINGHOUSE ELEC. CORP, OCEANIC DIV, Annapolis, MD (Dr. H. Newman and Dr. H. L. Price)	2
WESTINGHOUSE ELEC. CORP, Waltham, MA (D. Bennett)	1
WILSON JAMES H., San Clemente, CA	1
WOODS HOLE OCEANOGRAPHIC INSTITUTION (Dr. R. Spindel and Dr. E. Weinstein, Library)	3
YALE UNIV. (Library, Prof. P. Schultheiss and Prof. F. Tuteur)	3